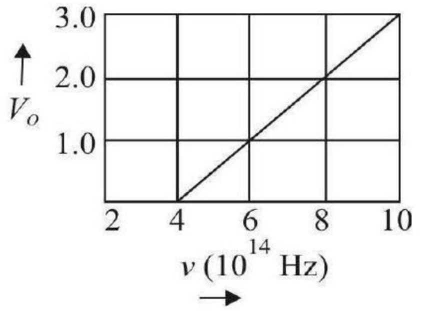
Dual Nature of Radiation and Matter

1. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of (2) The work function of the metal (in eV) is
(Energy of photon $= \frac{1240}{\lambda(\text{ in nm})} \text{eV}$)

2. The magnetic field associated with a light wave is given at the origin by
$B = B_0[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct]$.
If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy (in eV) of the photoelectrons?
$(c = 3 \times 10^8 \text{ ms}^{-1}, h = 6.6 \times 10^{-34} \text{ J} - \text{s})$

3. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 mW/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photo electrons. The number of emitted photo electrons per second is
$[1eV = 1.6 \times 10^{-19}$ J]

4. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function (in eV) of sodium, from the data plotted in the figure, will be :
(Given : Planck's constant $(h) = 6.63 \times 10^{-34}$Js, electron charge $e = 1.6 \times 10^{-19}$C)



5. If the de-Broglie wavelength of an electron is equal to 10^{-3} times the wavelength of a photon of frequency 6×10^{14} Hz, then the speed (in m/s) of electron is equal to :
(Speed of light $= 3 \times 10^8$ m/s)
Planck's constant $= 6.63 \times 10^{-34}$ J.s
Mass of electron $= 9.1 \times 10^{-31}$ kg)

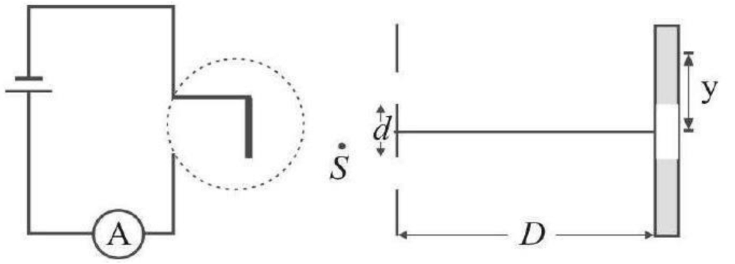
6. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential (in V) is

$$\left(\frac{hc}{e} = 1240 \text{ nm} - \text{V}\right)$$

7. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50v Another particle B of mass '4 m' and charge' q' is accelerated by a potential diffence of
2500 V. The ratio of de-Broglie wavelength $\frac{\lambda_A}{\lambda_B}$ is

8. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength (in nm) of photons emitted by mercury atoms is

9. In the arrangement shown in figure, $y = 1.0$ mm, $d = 0.24$ mm and $D = 1.2$ m. The work function of the material of the emitter is 2.2 eV. If stopping potential is $0.3x$, then value of x (in V) is



10. The electric field of light wave is given as

$$\vec{E} = 10^3 \cos\left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right) \hat{x} \frac{N}{C}$$

This light falls on a metal plate of work function 2 eV. The stopping potential (in V) of the photo-electrons is:

Given, $\mathrm{E(ineV)} = \frac{12375}{\lambda(\text{inÅ})}$

11. In a photoelectric effect experiment the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy (in eV) of emitted electrons will be:

Given E (in eV) $= \frac{1237}{\lambda(\text{innm})}$

12. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is : [Given Planck's constant $\mathrm{h} = 6.6 \times 10^{-34}$Js, speed of light $c = 3.0 \times 10^8$ m/s]

13. A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wavelength (in nm) of the light.

14. Light of wavelength 180 nm ejects photoelectron from a plate of a metal whose work function is 2 eV. If a uniform magnetic field of 5×10^{-5} T is applied parallel to plate, what would be the radius (in metre) of the path followed by electrons ejected normally from the plate with maximum energy?

15. A 100 W point source emits monochromatic light of wavelength 6000Å. What is the total number of photons emitted by the source per second?

SOLUTIONS

1. **(1.8)** From Einstein's photoelectric equation,

$$\frac{hc}{l_1} = f + \frac{1}{2}m(2v)^2 \quad ....(i)$$

and $$\frac{hc}{l_2} = f + \frac{1}{2}mv^2 \quad ....(ii)$$

As per question, maximum speed of photoelectrons in two cases differ by a factor 2

From eqn. (i) & (ii)

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\phi \quad \Rightarrow \phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right) = 1.8\,\text{eV}$$

2. **(7.7)** According to question, there are two EM waves with different frequency,

$B_1 = B_0 \sin(\pi \times 10^7 c)t$

and $B_2 = B_0 \sin(2\pi \times 10^7 c)t$

To get maximum kinetic energy we take the photon with higher frequency

using, $B = B_0 \sin \omega t$ and $\omega = 2\pi v \Rightarrow v = \frac{\omega}{2\pi}$

$B_1 = B_0 \sin(\pi \times 10^7 c)t \Rightarrow v_1 = \frac{10^7}{2} \times c$

$B_2 = B_0 \sin(2\pi \times 10^7 c)t \Rightarrow v_2 = 10^7 c$

where c is speed of light $c = 3 \times 10^8$ m/s

Clearly, $v_2 > v_1$

so KE of photoelectron will be maximum for photon of higher energy.

$v_2 = 10^7 c$ Hz

$hv = \phi + KE_{max}$

energy of photon

$E_{ph} = hv = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$

$E_{ph} = 6.6 \times 3 \times 10^{-19}$ J

$$= \frac{6.6 \times 3 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 12.375\,\text{eV}$$

$KE_{max} = E_{ph} - \phi$

$= 12.375 - 4.7 = 7.675\ \text{eV} \approx 7.7\ \text{eV}$

3. **(10^{11})** Using, intensity $I=\frac{nE}{At}$

n = no. of photoelectrons

$$\Rightarrow 16\times10^{-3}=\left(\frac{n}{t}\right)\times\frac{10\times1.6\times10^{-19}}{10^{-4}} \text{ or, } \frac{n}{t}=10^{12}$$

So, effective number of photoelectrons ejected per unit time = $10^{12}\times 10/100 = 10^{11}$

4. **(1.66)** $f_0 = 4\times10^{14}$ Hz

$W_0 = hf_0 = 6.63\times10^{-34}\times(4\times10^{14})$ J

$$=\frac{(6.63\times10^{-34})\times(4\times10^{14})}{1.6\times10^{-19}}$$

= 1.66 eV

5. **(1.45×10^6)** de-Broglie wavelength,

$$\lambda=\frac{h}{mv}=10^{-3}\left(\frac{3\times10^{8}}{6\times10^{14}}\right) \quad \left[\because \lambda=\frac{c}{v}\right]$$

$$v=\frac{6.63\times10^{-34}\times6\times10^{14}}{9.1\times10^{-31}\times3\times10^{5}}$$

$v = 1.45\times10^6$ m/s

6. **(1)** Let ϕ = work function of the metal,

$$\frac{hc}{\lambda_1}=\phi+eV_1 \quad ......(i)$$

$$\frac{hc}{\lambda_2}=\phi+eV_2 \quad ......(ii)$$

Sutracting (ii) from (i) we get

$$hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)=e(V_1-V_2)$$

$$\Rightarrow V_1-V_2=\frac{hc}{e}\left(\frac{\lambda_2-\lambda_1}{\lambda_1\cdot\lambda_2}\right) \quad \begin{bmatrix}\lambda_1=300\,nm\\ \lambda_2=400\,nm\\ \frac{hc}{e}=1240\,nm-V\end{bmatrix}$$

$$=(1240\,nm-v)\left(\frac{100nm}{300\,nm\times400\,nm}\right)$$

$=1.03\,V\approx1V$

7. **(14.14)** de-Broglie wavelength (λ) is given by

$K=qV$

$$\lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}=\frac{h}{\sqrt{2mqV}}\left(\because p=\sqrt{2mK}\right)$$

Substituting the values we get

$$\therefore \frac{\lambda_A}{\lambda_B}=\frac{\sqrt{2m_Bq_BV_B}}{\sqrt{2m_Aq_AV_A}}=\sqrt{\frac{4m.q.2500}{m.q.50}}$$

$$=2\sqrt{50}=2\times7.07=14.14$$

8. **(250)** Using, wavelength, $\lambda = \frac{12375}{\Delta E}$

or, $\lambda = \frac{12375}{4.9} \simeq 250\text{nm}$

9. **(3)** If λ is the wavelength emitted by the source S, then fringe width,

$$\beta = 2y = \frac{D\lambda}{d}$$

$$\therefore \quad \lambda = \frac{2yd}{D}$$

$$= \frac{2\times(1\times10^{-3})\times(0.24\times10^{-3})}{1.2}$$

$$= 0.4\times10^{-6}\,\text{m}$$

By Einstein equation,

$$hf = W_0 + \text{eV}$$

$$\text{or} \quad \frac{hc}{\lambda} = W_0 + eV$$

$$\therefore \quad V = \frac{hc}{e\lambda} - \frac{W_0}{e}$$

$$= \frac{(6.63\times10)^{-34}\times(3\times10^8)}{(1.6\times10^{-19})\times(0.4\times10^{-6})} - \frac{2.2\times1.6\times10^{-19}}{1.6\times10^{-19}}$$

$$= 0.9\text{V}$$

10. **(0.48)** Here $\omega = 2\pi \times 6 \times 10^{14}$

$\Rightarrow f = 6 \times 10^{14}$ Hz

Wavelength

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 0.5\times10^{-6}m = 5000\text{Å}$$

Given $E = \frac{12375}{5000} = 2.48\,eV$

Using $E = W + eV_s$

$\Rightarrow 2.48 = 2 + eV_s$

or $V_s = 0.48$ V

11. (1.5) $KE_{max} = E - f_0$

(where E = energy of incident light f_0 = work

function) $= \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$$= 1237\left[\frac{1}{260} - \frac{1}{380}\right]$$

$$= \frac{1237\times120}{380\times260} = 1.5\,eV$$

12. (5 × 10^{15}) Energy of photon (E) is given by

$$E = \frac{hc}{\lambda}$$

Number of photons of wavelength λ emitted in *t* second from laser of power P is given by

$$n = \frac{Pt\lambda}{hc}$$

$$\Rightarrow \quad n = \frac{2\times\lambda}{hc} = \frac{2\times10^{-3}\times5\times10^{-7}}{2\times10^{-25}} \quad (\because t = 1\text{S})$$

$$\Rightarrow \quad n = 5 \times 10^{15}$$

13. (400) If E is the energy of each photon, then

$$nE = P$$

$$\therefore \quad E = \frac{P}{n} = \frac{200}{4\times10^{20}} = 50\times10^{-20} J$$

If λ is the wavelength of light, then

$$E = \frac{hc}{\lambda}$$

$$\therefore \quad \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34})\times(3\times10^{8})}{500\times10^{-20}}$$

$$\simeq 400\,\text{nm}$$

14. (0.149) If v_{max} is the speed of the fastest electron emitted from the metal surface, then

$$\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv_{max}^2$$

$$\frac{(6.63\times10^{-34})\times(3\times10^{8})}{(180\times10^{-9})}$$

$$= 2\times(1.6\times10^{-19}) + \frac{1}{2}(9.1\times10^{-31})v_{max}^2$$

$$\therefore \quad v = 1.31 \times 10^{6}\ \text{m/s}$$

The radius of the electron is given by

$$r = \frac{mv}{qB} = \frac{(9.1\times10^{-31})\times(1.31\times10^{6})}{(1.6\times10^{-19})\times(5\times10^{-9})}$$

$$= 0.149\,\text{m}$$

15. (3× 10^{20}) The energy of each photon

$$E = \frac{hc}{\lambda} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{6000\times10^{-10}}$$

$$= 3.3 \times 10^{-19}\ \text{J.}$$

The number of photon emitted per second

$$n = \frac{\text{Power}}{E} = \frac{100}{3.3\times10^{-19}} = 3\times10^{20}$$